Sec 3.2 – Basic Trigonometric Equations
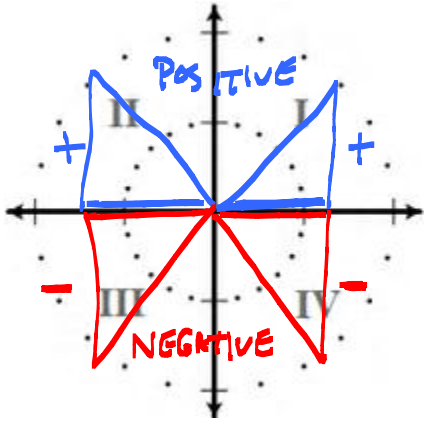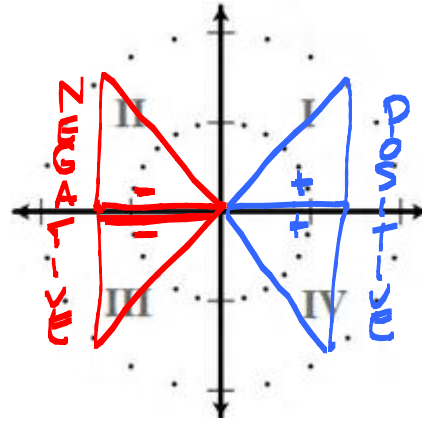## Trigonometric Functions   Name:

It may help to first review where each of the main trigonometric functions are positive and negative.
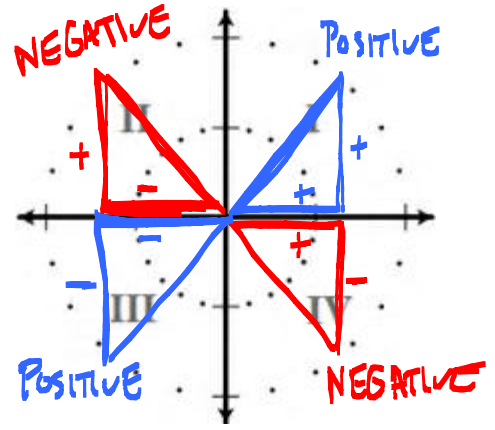**Indicate the quadrants where each function, is positive and negative.**

1. $y = sin(x)$

2. $y = cos(x)$

3. $y = tan(x)$

Using special right triangles find all the solutions of θ to the equations in <u>degrees</u> between 0° and 360°

4. $sin(\theta) = \frac{\sqrt{3}}{2}$ ←OP ←HYP   **60° or 120**

$\cancel{sin}^{-1}(sin(\theta)) = sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$

ONLY PROVIDES ONE ANS. → $\theta = 60°$

sin⁻¹(√(3)/2) = 60

5. $\frac{2\,sin(\theta)}{2} = \frac{-1}{2}$ **210° or 330°**

$sin(\theta) = \frac{-1}{2}$ ←OPP = -1 ←HYP = 2

sin⁻¹(-1/2) = -30

6. $\sqrt{2}\,cos(\theta) - 1 = 0$ **45° or 315°**
      +1   +1

$\frac{\sqrt{2}\,cos(\theta)}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

cos⁻¹(1/√(2)) = 45

$cos(\theta) = \frac{1}{\sqrt{2}}$ ←ADJ = 1 ←HYP = √2

7. $2\,cos(\theta) + \sqrt{3} = 0$ **150° or 210°**
      $-\sqrt{3}$ $-\sqrt{3}$

$\frac{2\,cos(\theta)}{2} = \frac{-\sqrt{3}}{2}$
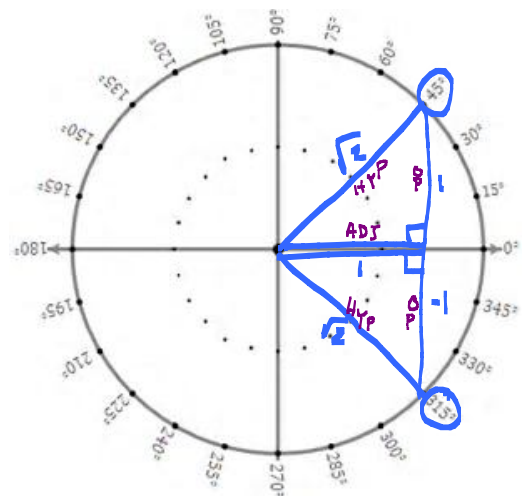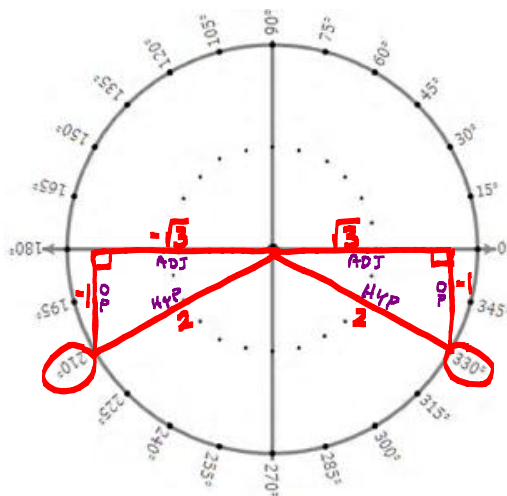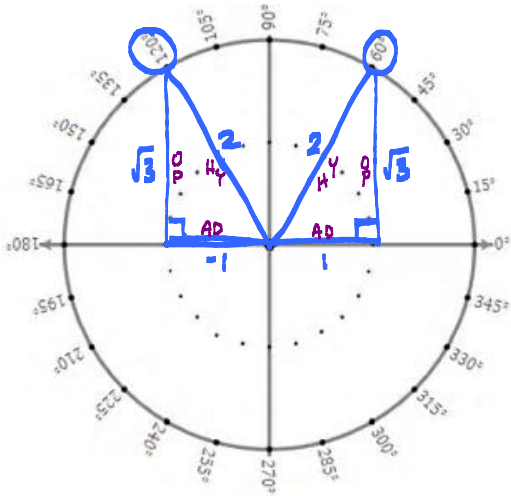
$cos(\theta) = \frac{-\sqrt{3}}{2}$ ←ADJ ←HYP

cos⁻¹(-√(3)/2) = 150

FOR **SINE**, USE THE CALCULATOR'S ANSWER, θ, AND TO FIND THE OTHER ANSWER USE 180 - θ.

FOR **COSINE**, USE THE CALCULATOR'S ANSWER, θ, AND TO FIND THE OTHER ANSWER USE 360 - θ.

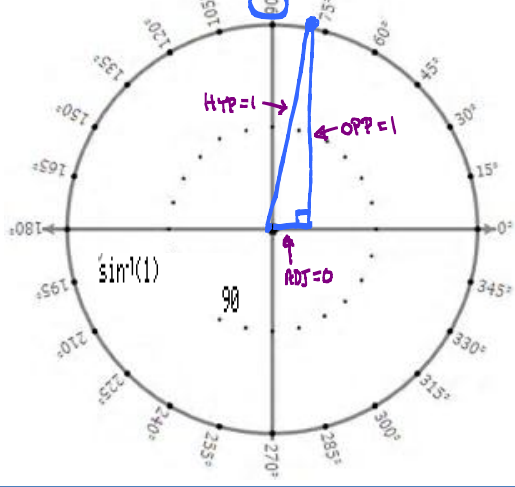FOR **TANGENT**, USE THE CALCULATOR ANSWER, θ, AND FOR THE OTHER ANSWER USE 180 + θ OR θ - 180.

Using the circle find all the solutions of θ to the equations in <u>degrees</u> within the range $0° \le \theta < 360°$.

8. $\dfrac{\sqrt{3}\,\tan(\theta)}{\sqrt{3}} = \dfrac{-1}{\sqrt{3}}$ 150° or 330°

$\tan\theta = \dfrac{-1}{\sqrt{3}}$ ← OPP = −1, ← ADJ = $\sqrt{3}$

tan⁻¹(−1/√(3)) = −30
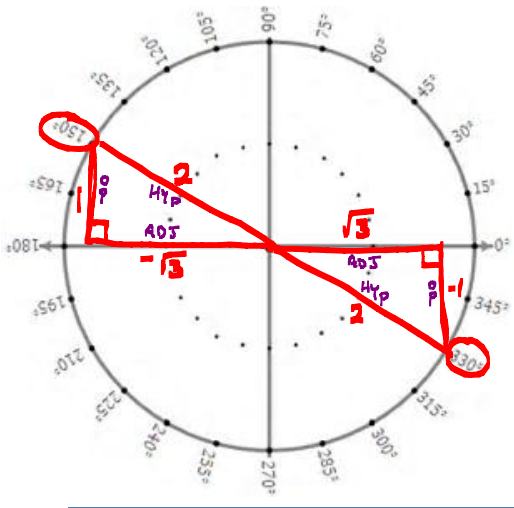
9. $\sin(\theta) = 1$ ← OPP, ← HYP   90°

THIS IS A QUADRANTAL ANGLE. So, CONSIDER DRAWING A TRIANGLE JUST BEFORE IT COLLAPSES AT 90° AND THEN WRITE WHAT THE MEASURES OF THE SIDES WOULD BE AT 90°

HYP = 1 → ← OPP = 1
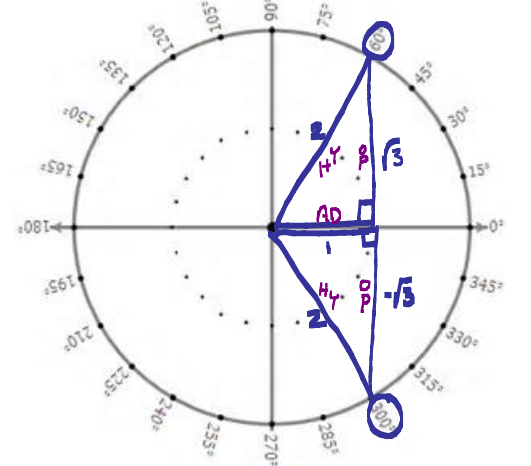
ADJ = 0

sin⁻¹(1) = 90

10. $\cos(\theta) + 1 = 1$ −1 −1   90° or 270°

$\cos(\theta) = \dfrac{0}{1}$ ← ADJ, ← HYP

cos⁻¹(0) = 90

HYP = 1 → ← OPP = 1

ADJ = 0 ↓

↑ ADJ = 0

OPP = −1 →
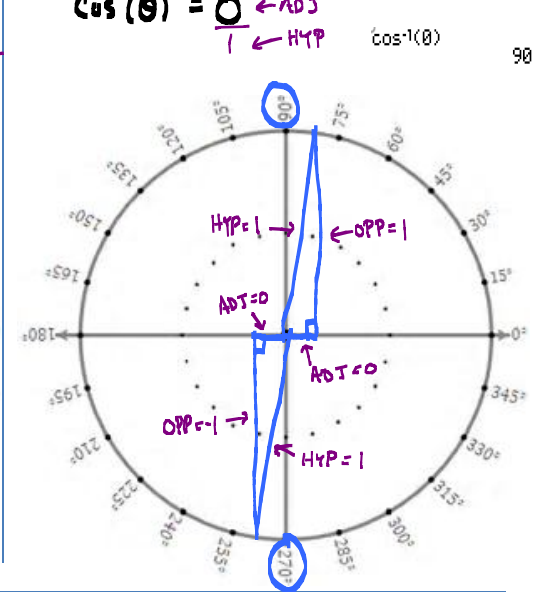
← HYP = 1

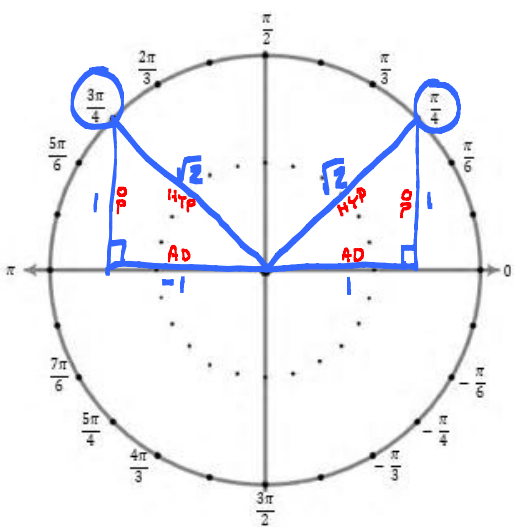

***11.** $2\cos(\theta)\sin(\theta) - \sin(\theta) = 0$ (FACTOR $\sin(\theta)$ OUT.)   0°, 180°, 60°, AND 300°

$(\sin(\theta))(2\cos(\theta) - 1) = 0$

USE THE ZERO PRODUCT PROPERTY

$\sin(\theta) = \dfrac{0}{1}$ ← OPP, ← HYP

QUADRANTAL ANGLE. HAPPENS AT 0° AND 180°

$2\cos(\theta) - 1 = 0$

$+1 \quad +1$

$\dfrac{2\cos(\theta)}{2} = \dfrac{1}{2}$

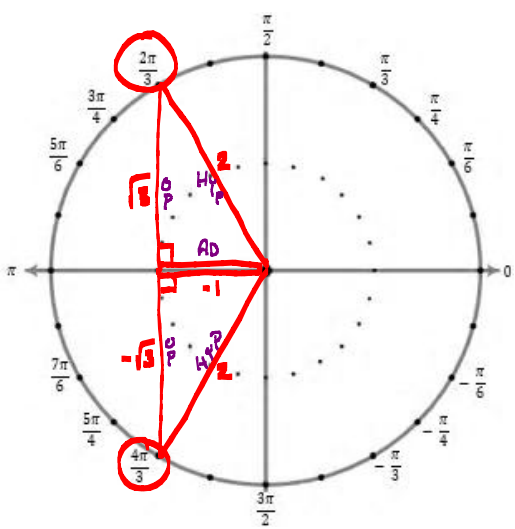$\cos(\theta) = \dfrac{1}{2}$ ← ADJ, ← HYP

Using the circle find all the solutions of x to the equations in <u>radians</u> within the range $0 \le x < 2\pi$

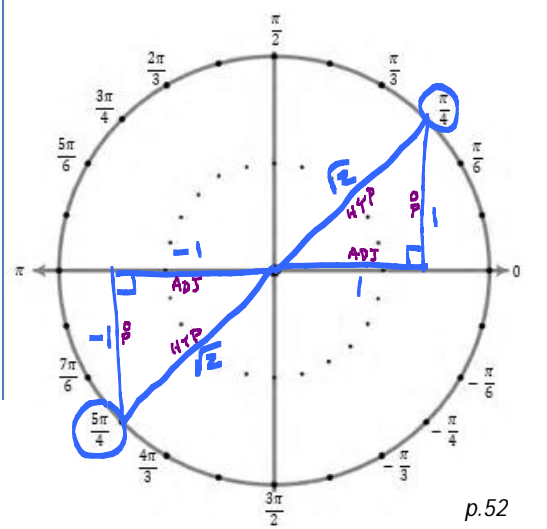**12.** $\sin(x) = \dfrac{\sqrt{2}}{2} = \dfrac{1}{\sqrt{2}}$ ← OPP, ← HYP $\dfrac{\pi}{4}$ or $\dfrac{3\pi}{4}$

sin⁻¹(√(2)/2)/π▶Frac = 1/4

13. $\cos(x) = -\dfrac{1}{2}$ ← ADJ = −1, ← HY = 2 $\dfrac{2\pi}{3}$ or $\dfrac{4\pi}{3}$

cos⁻¹(−1/2)/π▶Frac = 2/3

14. $\tan(x) = 1$ ← OPP, ← ADJ $\dfrac{\pi}{4}$ or $\dfrac{5\pi}{4}$

tan⁻¹(1)/π▶Frac = 1/4

Using the circle find all the solutions of *x* to the equations in <u>radians</u> within the range 0 ≤ *x* < 2π

15. $2\sin(x) = 1$

$\boxed{\frac{\pi}{6} \text{ or } \frac{5\pi}{6}}$

$\sin(x) = \frac{1}{2} \leftarrow \text{OPP} \leftarrow \text{HYP}$

sin⁻¹(1/2)/π►Frac
1/6

16. $\cos(x) = -1 \leftarrow \text{ADJ} \leftarrow \text{HYP}$

$\boxed{\pi}$

QUADRANTAL ANGLE

cos⁻¹(-1)/π►Frac
1

17. $\tan(x) = undefined$

$\boxed{\frac{\pi}{2} \text{ or } \frac{3\pi}{2}}$

$= \frac{1}{0} \begin{smallmatrix}\leftarrow \text{OPP}\\\leftarrow \text{ADJ}\end{smallmatrix}$ OR $\frac{-1}{0} \begin{smallmatrix}\leftarrow \text{OPP}\\\leftarrow \text{ADJ}\end{smallmatrix}$
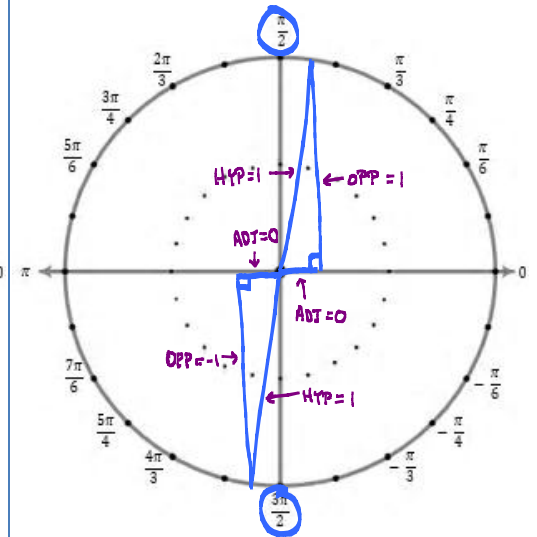
QUADRANTAL ANGLE

Using the circle find all the solutions of *θ* to the equations in <u>degrees</u> within the range 0° ≤ *θ* < 360°.

18. $\sin(\theta) = 0.78$ ≈ $\boxed{51.26° \text{ OR } 128.74°}$

$\theta_1 = \sin^{-1}(0.78) \leftarrow \text{ONE ANS.}$

$\theta_1 \approx 51.2606°$ sin⁻¹(0.78)
51.2605754

$\theta_2 = 180 - 51.2606$ 180−51.2605754
128.7394246

THIS WILL WORK WITH SINE.

$\theta_2 \approx 128.7394°$
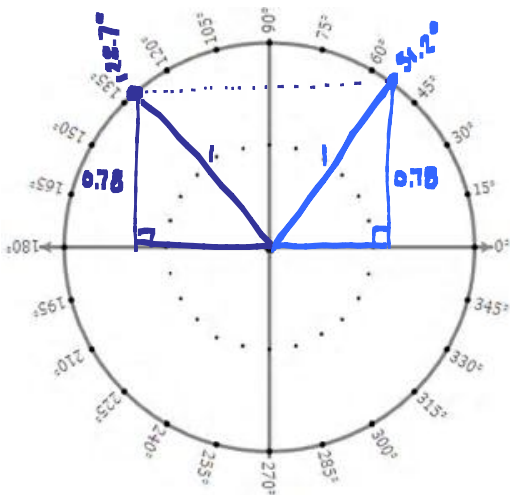
19. $\sin(\theta) = -0.41$

$\boxed{204.20° \text{ OR } 335.80°}$

$\theta_0 = \sin^{-1}(-0.41)$

$\theta_0 \approx -24.20$

sin⁻¹(-0.41)
-24.2048348
Ans+360
335.7951652

ADD 360° TO FIND COTERMINAL ANGLE BETWEEN 0° AND 360°

$\theta_1 \approx 360 + -24.20 = 335.80°$

$\theta_2 \approx 180 - -24.20 = 204.20°$
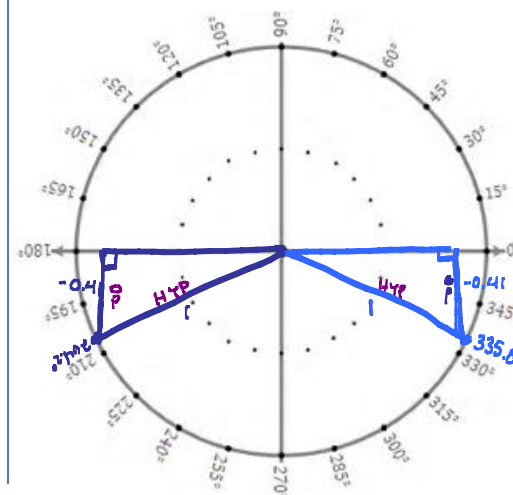
20. $\cos(\theta) = 0.32$

$\boxed{71.34° \text{ OR } 288.66°}$

$\theta_1 = \cos^{-1}(0.32)$

$\theta_1 \approx 71.34°$

cos⁻¹(.32)
71.33707512

$\theta_2 \approx 360 - 71.34°$

THIS WORKS FOR COSINE

$\theta_2 \approx 288.66°$
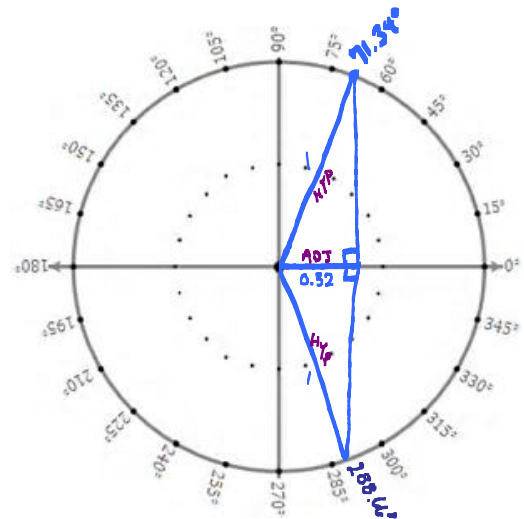
Using the circle find all the solutions of θ to the equations in <u>degrees</u> within the range $0° \le \theta < 360°$.

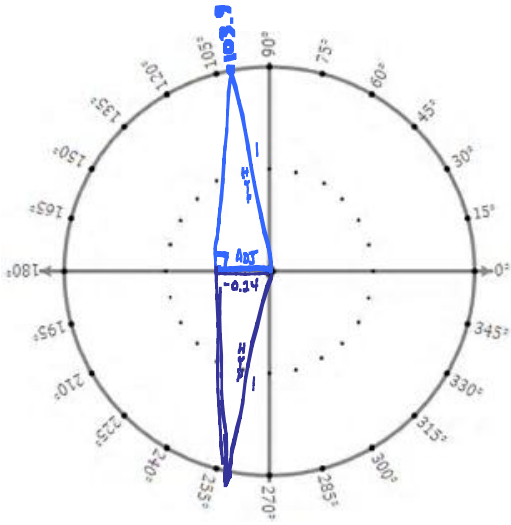**21.** $cos(\theta) = -0.24$ | **103.89** or **256.11°**

$\theta_1 = Cos^{-1}(-0.24)$
$\theta_1 \approx 103.89°$

$\theta_2 \approx 360 - 103.89$
$\theta_2 \approx 256.11°$

cos⁻¹(-0.24)
103.8865404

360-103.8865404
256.1134596

22. $tan(\theta) = 1.41$

$\theta_1 = tan^{-1}(1.41)$
$\theta_1 \approx 54.65°$

$\theta_2 = \underbrace{180 + 54.65°}_{\text{THIS WILL WORK FOR TANGENT}}$

$\theta_2 = 234.65°$

tan⁻¹(1.41)
54.6549771

180+54.6549771
234.6549771

23. $tan(\theta) = -0.87$
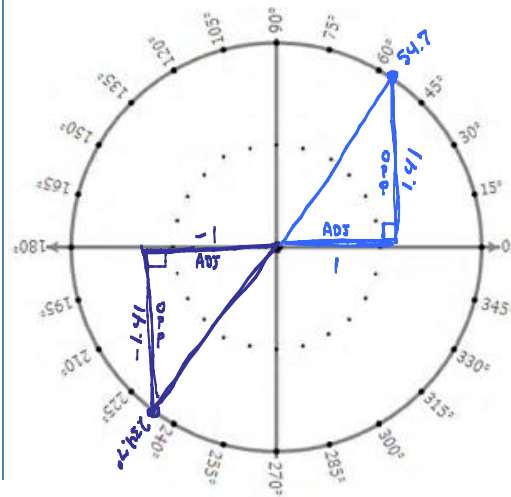
$\theta_0 = tan^{-1}(-0.87)$
$\theta_0 \approx -41.02$ ← NOT BETWEEN 0° AND 360°

$\theta_1 \approx 360 + -41.02$
$\theta_1 \approx 318.98°$

$\theta_2 \approx 318.98 - 180$
$\theta_2 \approx 138.98$

tan⁻¹(-0.87)
-41.02326902

360+ -41.0232690
318.976731

318.976731-180
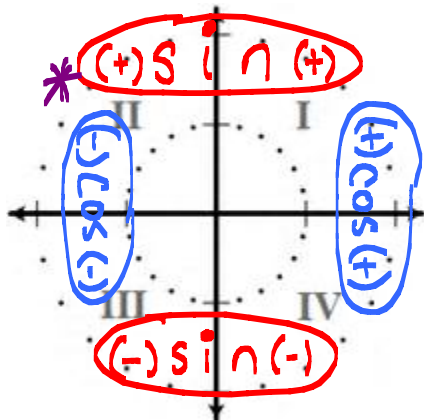138.976731
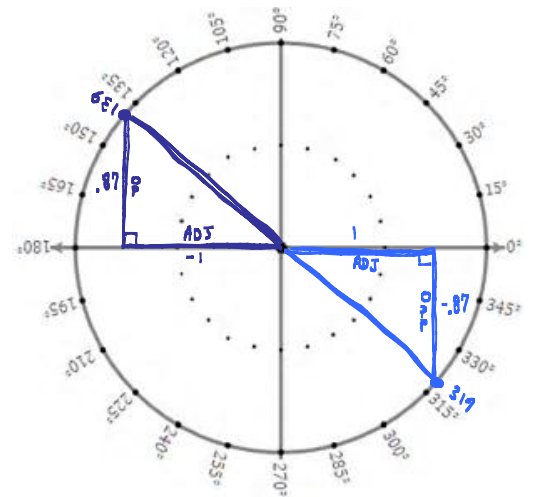
Which quadrant, if any, is described by the given constraints?

24. $sin(x) > 0$ and $cos(x) < 0$
Pos (+)   NEG (-)

QUADRANT II

25. $cos(x) < 0$ and $tan(x) < 0$
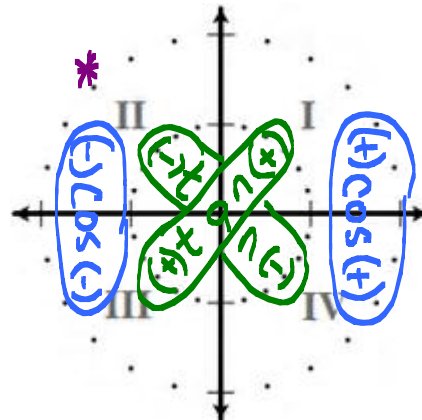NEG (-)   NEG (-)

QUADRANT II

26. $sin(x) < 0$ and $cos(x) > 0$
NEG (-)   Pos (+)

QUADRANT IV